Chapter 23
Mirrors and Lenses

Problem Solutions

23.1 If you stand 40 cm in front of the mirror, the time required for light scattered from your face to travel to the mirror and back to your eye is

\[ \Delta t = \frac{2d}{c} = \frac{2 \cdot 0.40 \text{ m}}{3.0 \times 10^8 \text{ m/s}} = 2.7 \times 10^{-9} \text{ s} \]

Thus, the image you observe shows you \(~10^{-9} \text{ s younger}\) than your current age.

23.2 (a) With the palm located 1.0 m in front of the nearest mirror, that mirror forms an image, \( I_{p1} \), of the palm located \([1.0 \text{ m behind the nearest mirror}]\).

(b) The farthest mirror forms an image, \( f > 0 \), of the back of the hand located 2.0 m behind this mirror and 5.0 m in front of the nearest mirror. This image serves and an object for the nearest mirror, which then forms an image, \( f = |f| \), of the back of the hand located \([5.0 \text{ m behind the nearest mirror}]\).

(c) The image \( I_{p1} \) (see part a) serves as an object located 4.0 m in front of the farthest mirror, which forms an image \(|p| > |f|\) of the palm, located 4.0 m behind that mirror and 7.0 m in front of the nearest mirror. This image then serves as an object for the nearest mirror which forms an image \( I_{p3} \) of the palm, located \([7.0 \text{ m behind the nearest mirror}]\).

(d) Since all images are located behind the mirror, all are virtual images.

23.3 (1) The first image in the left-hand mirror is 5.00 ft behind the mirror, or \([10.0 \text{ ft from the person}]\).
(2) The first image in the right-hand mirror serves as an object for the left-hand mirror. It is located 10.0 ft behind the right-hand mirror, which is 25.0 ft from the left-hand mirror. Thus, the second image in the left-hand mirror is 25.0 ft behind the mirror, or \[ p_2 = 40.0 \text{ cm} - q_1 = +10.0 \text{ cm} \]

(3) The first image in the left-hand mirror serves as an object for the right-hand mirror. It is located 20.0 ft in front of the right-hand mirror and forms an image 20.0 ft behind that mirror. This image then serves as an object for the left-hand mirror. The distance from this object to the left-hand mirror is 35.0 ft. Thus, the third image in the left-hand mirror is 35.0 ft behind the mirror, or \[ q_2 = \frac{p_3 f_2}{p_2 - f_2} = \frac{10.0 \text{ cm} \times 15.0 \text{ cm}}{10.0 \text{ cm} - 15.0 \text{ cm}} = -30.0 \text{ cm} \]

23.4 The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror.

The image of the choir is

\[ M_2 = -\frac{q_2}{p_2} = -\frac{-30.0 \text{ cm}}{10.0 \text{ cm}} = +3.00 \]

from the organist. Using similar triangles, gives

\[ \frac{h'}{0.600 \text{ m}} = \frac{6.10 \text{ m}}{0.800 \text{ m}} \]

or \[ h' = 0.600 \text{ m} \left( \frac{6.10 \text{ m}}{0.800 \text{ m}} \right) = 4.58 \text{ m} \]

23.5 In the figure at the right, \( \theta = \theta \) since they are vertical angles formed by two intersecting straight lines. Their complementary angles are also equal or \( p_i = +15.0 \text{ cm} \). The right triangles

\[ q_i = \frac{p_i f_i}{p_i - f_i} = \frac{15.0 \text{ cm} \times 10.0 \text{ cm}}{15.0 \text{ cm} - 10.0 \text{ cm}} = +30.0 \text{ cm} \]

and \( PQR \) have the common side \( QR \) and are then congruent by the angle-side-angle theorem. Thus, the corresponding sides \( PQ \) and \( q_2 = -10.0 \text{ cm} + p_i = -10.0 \text{ cm} + 15.0 \text{ cm} = -25.0 \text{ cm} \) are equal, or the image is as far behind the mirror as the object is in front of it.
23.6 (a) Since the object is in front of the mirror,

\[ f_2 = \frac{p_2 q_2}{p_2 + q_2} = \frac{-20.0 \text{ cm}}{-20.0 \text{ cm} - 25.0 \text{ cm}} = -11.1 \text{ cm} \]. With the image behind the mirror, \( q < 0 \). The mirror equation gives the radius of curvature as

\[ \frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{1.00 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = \frac{10 - 1}{10.0 \text{ cm}} \]

or

\[ R = 2 \left( \frac{10.0 \text{ cm}}{9} \right) = +2.22 \text{ cm} \]

(b) The magnification is virtual.

23.7 (a) Since the mirror is concave, \( M > 0 \). Because the object is located in front of the mirror, \( p > 0 \). The mirror equation, \( \frac{1}{f} + \frac{1}{q} = \frac{2}{R} \) then gives the image distance as

\[ q = \frac{pR}{2p - R} = \frac{40.0 \text{ cm}}{2} \left( \frac{20.0 \text{ cm}}{40.0 \text{ cm} - 20.0 \text{ cm}} \right) = +13.3 \text{ cm} \]

Since \( q > 0 \), the image is located +13.3 cm in front of the mirror.

(b) \[ q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{14.0 \text{ cm}}{14.0 \text{ cm} - (-16.0 \text{ cm})} = -7.47 \text{ cm} \]

Because \( M_2 = \frac{q_2}{p_2} = \frac{-7.47 \text{ cm}}{14.0 \text{ cm}} = +0.533 \), \( M = M_1 M_2 = +2.00 \cdot +0.533 = +1.07 \) and since 7.47 cm in front of the second lens, \( h' = M h = +1.07 \cdot 1.00 \text{ cm} = 1.07 \text{ cm} \).
23.8 The lateral magnification is given by $M > 0$. Therefore, the image distance is upright.

The mirror equation: $\frac{1}{R} = \frac{1}{p} + \frac{1}{q}$ or virtual

gives $q_2 = -50.0 \text{ cm} - 31.0 \text{ cm} = -19.0 \text{ cm}$

The negative sign tells us that the surface is convex. The magnitude of the radius of curvature of the cornea is

$$p_2 = \frac{q_2 f_2}{q_2 - f_2} = \frac{-19.0 \text{ cm} - 20.0 \text{ cm}}{-19.0 \text{ cm} - 20.0 \text{ cm}} = +9.74 \text{ cm}$$

23.9 (a) For a convex mirror, the focal length is $q_1 = 50.0 \text{ cm} - 9.74 \text{ cm} = 40.3 \text{ cm}$, and with the object in front of the mirror, $p_1 = \frac{q_1 f_1}{q_1 - f_1} = \frac{40.3 \text{ cm} 
10.0 \text{ cm}}{40.3 \text{ cm} - 10.0 \text{ cm}} = +13.3 \text{ cm}$. The mirror equation, $13.3 \text{ cm}$, then gives

$$M = M_1 M_2 = \left( \frac{q_1}{p_1} \right) \left( \frac{-q_2}{p_2} \right) = \left( \frac{-40.3 \text{ cm}}{13.3 \text{ cm}} \right) \left( \frac{-19.0 \text{ cm}}{9.74 \text{ cm}} \right) = -5.90$$

With $M < 0$, the image is located inverted.

(b) The magnification is

$$M = \frac{h'}{h} = -\frac{q}{p} = -\frac{-7.50 \text{ cm}}{30.0 \text{ cm}} = +0.250$$

Since virtual and $M > 0$, the image is virtual and upright. Its height is

$$h' = Mh = 0.250 \times 2.0 \text{ cm} = 0.50 \text{ cm}$$
23.10 The image was initially upright but became inverted when Dina was more than 30 cm from the mirror. From this information, we know that the mirror must be concave because a convex mirror will form only upright, virtual images of real objects.

When the object is located at the focal point of a concave mirror, the rays leaving the mirror are parallel, and no image is formed. Since Dina observed that her image disappeared when she was about 30 cm from the mirror, we know that the focal length must be \( f \approx 30 \text{ cm} \). Also, for spherical mirrors, \( R = 2f \). Thus, the radius of curvature of this concave mirror must be \( R \approx 60 \text{ cm} \).

23.11 The magnified, virtual images formed by a concave mirror are upright, so \( M > 0 \).

Thus, \( M = \frac{q}{p} = \frac{h'}{h} = \frac{5.00 \text{ cm}}{2.00 \text{ cm}} = +2.50 \), giving

\[ q = -2.50 \quad p = -2.50 \quad +3.00 \text{ cm} = -7.50 \text{ cm} \]

The mirror equation then gives,

\[ \frac{1}{f} = \frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{3.00 \text{ cm}} - \frac{1}{7.50 \text{ cm}} = \frac{2.50 - 1}{7.50 \text{ cm}} \]

or \( f = \frac{7.50 \text{ cm}}{1.50} = 5.00 \text{ cm} \).
23.12  Realize that the magnitude of the radius of curvature, $|R|$, is the same for both sides of the hubcap. For the convex side, $R = -|R|$; and for the concave side, $R = +|R|$. The object distance $p$ is positive (real object) and has the same value in both cases. Also, we write the virtual image distance as $q = -|q|$ in each case. The mirror equation then gives:

For the convex side, \[\frac{1}{-|q|} = \frac{2}{-|R|} - \frac{1}{p}\] or \[|q| = \frac{|R|p}{|R| + 2p}\] \[\text{[1]}\]

For the concave side, \[\frac{1}{-|q|} = \frac{2}{|R|} - \frac{1}{p}\] or \[|q| = \frac{|R|p}{|R| - 2p}\] \[\text{[2]}\]

Comparing Equations [1] and [2], we observe that the smaller magnitude image distance, $|q| = 10.0 \text{ cm}$, occurs with the convex side of the mirror. Hence, we have

\[
\frac{1}{-10.0 \text{ cm}} = \frac{2}{-|R|} - \frac{1}{p}
\] \[\text{[3]}\]

and for the concave side, $|q| = 30.0 \text{ cm}$ gives

\[
\frac{1}{-30.0 \text{ cm}} = \frac{2}{|R|} - \frac{1}{p}
\] \[\text{[4]}\]

(a) Adding Equations [3] and [4] yields \[\frac{2}{p} = \frac{3+1}{30.0 \text{ cm}}\] or \[p = \pm 15.0 \text{ cm}\]

(b) Subtracting [3] from [4] gives \[\frac{4}{|R|} = \frac{3-1}{30.0 \text{ cm}}\] or \[|R| = \pm 60.0 \text{ cm}\]

23.13  The image is upright, so $M > 0$, and we have

\[M = -\frac{q}{p} = +2.0, \text{ or } q = -2.0 p = -2.0 \times 25 \text{ cm } = -50 \text{ cm}\]

The radius of curvature is then found to be

\[\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{25 \text{ cm}} - \frac{1}{50 \text{ cm}} = \frac{2-1}{50 \text{ cm}}, \text{ or } R = 2 \left(\frac{0.50 \text{ m}}{+1}\right) = 1.0 \text{ m}\]
23.14 (a) Your ray diagram should be carefully drawn to scale and look like the diagram

![Ray Diagram]

(b) From the mirror equation with \( \frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50 \text{ cm}} \) and \( q_2 = 7.50 \text{ cm} \), the image distance is

\[
\frac{1}{f} = n - 1 \left( \frac{1}{R_1} - \frac{1}{R_2} \right)
\]

and the magnification is \( \frac{1}{5.00 \text{ cm}} = n - 1 \left( \frac{1}{9.00 \text{ cm}} - \frac{1}{-11.0 \text{ cm}} \right) \). Thus, you should find that the image is

upright, located 6.00 cm behind the mirror, six-tenths the size of the object.
23.15 The focal length of the mirror may be found from the given object and image distances as
\[ q_1 = \frac{p_1 f}{p_1 - f} = \frac{8.00 \text{ cm} - 5.00 \text{ cm}}{8.00 \text{ cm} - 5.00 \text{ cm}} = +13.3 \text{ cm}, \text{ or} \]
\[ f = \frac{p q}{p + q} = \frac{152 \text{ cm} - 18.0 \text{ cm}}{152 \text{ cm} + 18.0 \text{ cm}} = +16.1 \text{ cm} \]

For an upright image twice the size of the object, the magnification is
\[ q_2 = \frac{p_2 R}{2 p_2 - R} = \frac{6.67 \text{ cm} - 8.00 \text{ cm}}{2 \times 6.67 \text{ cm} + 8.00 \text{ cm}} = +10.0 \text{ cm} \text{ giving } q = -2.00 p \]

Then, using the mirror equation again, \( 1/p + 1/q = 1/f \) becomes
\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{2} \frac{1}{2} = \frac{2-1}{2.00} = \frac{1}{f} \]
\[ \text{or} \quad p = \frac{f}{2.00} = \frac{16.1 \text{ cm}}{2.00} = 8.05 \text{ cm} \]

23.16 (a) The mirror is convex, so \( M < 0 \), and we have \( f = -|f| = -8.0 \text{ cm} \). The image is virtual, so \( p = \frac{q f}{q - f} \), or \( q = -|q| \). Since we also know that \( |q| = p/3 \), the mirror equation gives
\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{p} - \frac{3}{p} = \frac{1}{f} \text{ or } -\frac{2}{p} = \frac{1}{-8.0 \text{ cm}} \text{ and } p = +16 \text{ cm} \]

This means that we have \[ \text{a real object located 16 cm in front of the mirror} \].

(b) The magnification is \( M = -q/p = +|q|/p = +1/3 \). Thus, the image is \[ \text{upright} \] and one third the size of the object.

23.17 (a) We know that the object distance is \( p = +10.0 \text{ cm} \). Also, \( M > 0 \) since the image is upright, and \( |M| = 1/2 \) since the image is half the size of the object. Thus, we have
\[ M = -\frac{q}{p} = -\frac{q}{10.0 \text{ cm}} = +\frac{1}{2} \text{ or } f = \frac{-6.00 \text{ m}}{1.00 - 1.50} = -12.0 \text{ m} \]

and the image is seen to be located \( q = \frac{p f}{p - f} = \frac{10.0 \text{ m} - 12.0 \text{ m}}{10.0 \text{ m} + 12.0 \text{ m}} = -5.45 \text{ m} \).
(b) From the mirror equation, \(\frac{1}{p} + \frac{1}{q} = \frac{1}{f}\), we find the focal length to be

\[
f = \frac{pq}{p + q} = \frac{10.0 \text{ cm} \cdot (-5.00 \text{ cm})}{10.0 \text{ cm} + (-5.00 \text{ cm})} = -10.0 \text{ cm}
\]

23.18 (a) Since the mirror is concave, \(R > 0\), giving \(n_2 = 1.33\) and

\[
f = \frac{6.00 \text{ m} \cdot 1.33}{1.33 - 1.50} = -46.9 \text{ m}.
\]

Because the image is upright \((q = \frac{pf}{p - f} = \frac{10.0 \text{ m} \cdot (-46.9 \text{ m})}{10.0 \text{ m} + 46.9 \text{ m}} = -8.24 \text{ m})\) and three times the size of the object \((8.24 \text{ m to the left of the lens})\), we have

\[
M = -\frac{q}{p} = +3 \quad \text{and} \quad q = -3p
\]

The mirror equation then gives

\[
\frac{1}{p} - \frac{1}{3p} = \frac{2}{3p} = \frac{1}{12 \text{ cm}} \quad \text{or} \quad f = \frac{6.00 \text{ m}}{2.00 - 1.50} = +24.0 \text{ m}
\]

(b) The needed ray diagram, with the object 8.0 cm in front of the mirror, is shown below:

![Ray Diagram](image)

From a carefully drawn scale drawing, you should find that the image is

\[
q = \frac{pf}{p - f} = \frac{10.0 \text{ m} \cdot 24.0 \text{ m}}{10.0 \text{ m} - 24.0 \text{ m}} = -17.1 \text{ m}
\]

23.19 (a) An image formed on a screen is a real image. Thus, the mirror must be concave since, of mirrors, only concave mirrors can form real images of real objects.
(b) The magnified, real images formed by concave mirrors are inverted, so $M < 0$ and

$$M = \frac{q}{p} = -5,$$ or $p = \frac{q}{5} = \frac{5.0 \text{ m}}{5} = 1.0 \text{ m}$

The object should be $f > 0$

(a – revisited) The focal length of the mirror is

$$n_1 < n_2, \text{ or } f = \frac{5.0 \text{ m}}{6} = 0.83 \text{ m}$$

23.20 (a) From \(\frac{1}{p} + \frac{1}{q} = \frac{2}{R}\), we find \(q = \frac{R p}{2 p - R} = \frac{1.00 \text{ m} \cdot p}{2 p - 1.00 \text{ m}}\)

The table gives the image position at a few critical points in the motion. Between \(p = 3.00 \text{ m}\) and \(p = 0.500 \text{ m}\), the real image moves from 0.500 m to positive infinity. From \(q_{\text{mirror}} = p_{\text{mirror}} = 40.0 \text{ cm} - p_1 = 40.0 \text{ cm} - 16.7 \text{ cm} = +23.3 \text{ cm}\) to \(p = 0\), the virtual image moves from negative infinity to 0.

Note the “jump” in the image position as the ball passes through the focal point of the mirror.

(b) The ball and its image coincide when \(p = 0\) and when

$$\frac{1}{p} + \frac{1}{p} = \frac{2}{p} = \frac{2}{R}, \text{ or } n_1 = 1.55$$

From \(n_2\), with \(v_0y = 0\), the times for the ball to fall from \(p = 3.00 \text{ m}\) to these positions are found to be

$$t = \sqrt{\frac{2 \Delta y}{a_y}} = \sqrt{\frac{2 \cdot -2.00 \text{ m}}{-9.80 \text{ m/s}^2}} = 0.639 \text{ s}$$ and

$$\frac{1}{f_{\text{water}}} = \left(\frac{1.55 - 1.33}{1.33}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
23.21 From \( \frac{f_{\text{water}}}{f_{\text{air}}} = \left( \frac{1.33}{1.00} \right) \left( \frac{1.55 - 1.00}{1.55 - 1.33} \right) = 1.33 \left( \frac{0.55}{0.22} \right) \), with \( R \rightarrow \infty \), the image position is found to be

\[
q = -\frac{n_2}{n_1} p = \left( \frac{1.00}{1.309} \right) \left( 50.0 \, \text{cm} \right) = -38.2 \, \text{cm}
\]

or the virtual image is 38.2 cm below the upper surface of the ice.

23.22 The center of curvature of a convex surface is located behind the surface, and the sign convention for refracting surfaces (Table 23.2 in the textbook) states that \( R > 0 \), giving \( R = +8.00 \, \text{cm} \). The object is in front of the surface (\( p > 0 \)) and in air (\( n_1 = 1.00 \)), while the second medium is glass (\( n_2 = 1.50 \)). Thus, \( \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \) becomes

\[
\frac{1.00 + 1.50}{p} = \frac{1.50 - 1.00}{8.00 \, \text{cm}} \quad \text{and reduces to} \quad q = \frac{24.0 \, \text{cm}}{p - 16.0 \, \text{cm}}
\]

(a) If \( p = 20.0 \, \text{cm}, \quad q = \frac{24.0 \, \text{cm}}{20.0 \, \text{cm} - 16.0 \, \text{cm}} = +120 \, \text{cm} \)

(b) If \( p = 8.00 \, \text{cm}, \quad q = \frac{24.0 \, \text{cm}}{8.00 \, \text{cm} - 16.0 \, \text{cm}} = -24.0 \, \text{cm} \)

(c) If \( p = 4.00 \, \text{cm}, \quad q = \frac{24.0 \, \text{cm}}{4.00 \, \text{cm} - 16.0 \, \text{cm}} = -8.00 \, \text{cm} \)

(d) If \( p = 2.00 \, \text{cm}, \quad q = \frac{24.0 \, \text{cm}}{2.00 \, \text{cm} - 16.0 \, \text{cm}} = -3.43 \, \text{cm} \)

23.23 Since the center of curvature of the surface is on the side the light comes from, \( R < 0 \) giving \( R = -4.0 \, \text{cm} \). Then, \( \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \) becomes

\[
\frac{1.00}{q} = \frac{1.00 - 1.50}{-4.0 \, \text{cm}} = \frac{1.50}{4.0 \, \text{cm}} \quad \text{or} \quad q = -4.0 \, \text{cm}
\]

Thus, the magnification \( M = \frac{h'}{h} = -\left( \frac{n_1}{n_2} \right) \frac{q}{p} \), gives

\[
h' = -\left( \frac{n_2 q}{n_2 p} \right) h = -\frac{1.50}{1.00} \frac{-4.0 \, \text{cm}}{4.0 \, \text{cm}} = 2.5 \, \text{mm} = 3.8 \, \text{mm}
\]
23.24  For a plane refracting surface  \( R \to \infty \)

\[
\frac{n_1 + n_2}{p} + \frac{n_2 - n_1}{q} = \frac{n_2 - n_1}{R} \quad \text{becomes} \quad q = -\frac{n_2}{n_1} p
\]

(a) When the pool is full,  \( p = 2.00 \, \text{m} \) and

\[
q = -\left( \frac{1.00}{1.333} \right) 2.00 \, \text{m} = -1.50 \, \text{m}
\]

or the pool appears to be \( 1.50 \, \text{m} \) deep

(b) If the pool is half filled, then  \( p = 1.00 \, \text{m} \) and  \( q = -0.750 \, \text{m} \). Thus, the bottom of the pool appears to be 0.75 m below the water surface or \( 1.75 \, \text{m} \) below ground level.

23.25  As parallel rays from the Sun  \( \text{object distance, } p \to \infty \) enter the transparent sphere from air  \( n_1 = 1.00 \), the center of curvature of the surface is on the side the light is going toward (back side). Thus,  \( R > 0 \). It is observed that a real image is formed on the surface opposite the Sun, giving the image distance as  \( q = +2R \).

Then \( \frac{n_1 + n_2}{p} + \frac{n_2 - n_1}{q} = \frac{n_2 - n_1}{R} \) becomes \( 0 + \frac{n}{2R} = \frac{n - 1.00}{R} \)

which reduces to \( n = 2n - 2.00 \) and gives \( n = 2.00 \).
23.26 Light scattered from the bottom of the plate undergoes two refractions, once at the top of the plate and once at the top of the water. All surfaces are planes $R \to \infty$, so the image distance for each refraction is $q = -\frac{n_2}{n_1} p$. At the top of the plate,

$$q_{1B} = -\left( \frac{n_{\text{water}}}{n_{\text{glass}}} \right) p_{1B} = -\left( \frac{1.333}{1.66} \right) 8.00 \text{ cm} = -6.42 \text{ cm}$$

or the first image is 6.42 cm below the top of the plate. This image serves as a real object for the refraction at the top of the water, so the final image of the bottom of the plate is formed at

$$q_{2B} = -\left( \frac{n_{\text{air}}}{n_{\text{water}}} \right) p_{2B} = -\left( \frac{n_{\text{air}}}{n_{\text{water}}} \right) \left( 12.0 \text{ cm} + |q_{1B}| \right)$$

$$= -\left( \frac{1.00}{1.333} \right) 18.4 \text{ cm} = -13.8 \text{ cm} \text{ or } 13.8 \text{ cm} \text{ below the water surface.}$$

Now, consider light scattered from the top of the plate. It undergoes a single refraction, at the top of the water. This refraction forms an image of the top of the plate at

$$q_T = -\left( \frac{n_{\text{air}}}{n_{\text{water}}} \right) p_T = -\left( \frac{1.00}{1.333} \right) 12.0 \text{ cm} = -9.00 \text{ cm}$$

or 9.00 cm below the water surface.

The apparent thickness of the plate is then

$$\Delta y = |q_{2B}| - |q_T| = 13.8 \text{ cm} - 9.00 \text{ cm} = 4.8 \text{ cm}$$

23.27 In the drawing at the right, object $O$ (the jelly fish) is located distance $p$ in front of a plane water-glass interface. Refraction at that interface produces a virtual image $I'$ at distance $|q'|$ in front it. This image serves as the object for refraction at the glass-air interface. This object is located distance $p' = |q'| + t$ in front of the second interface, where $t$ is the thickness of the layer of glass. Refraction at the glass-air interface produces a final virtual image, $I$, located distance $|q|$ in front of this interface.
From \(n_1/p + n_2/q = (n_2 - n_1)/R\) with \(R \to \infty\) for a plane, the relation between the object and image distances for refraction at a flat surface is \(q = -\frac{n_2}{n_1} p\). Thus, the image distance for the refraction at the water-glass interface is \(q' = -\frac{n_g}{n_w} p\). This gives an object distance for the refraction at the glass-air interface of \(p' = (n_g/n_w)p + t\) and a final image position (measured from the glass-air interface) of

\[
q = -\frac{n_a}{n_g} p' = -\frac{n_a}{n_g} \left[ \left( \frac{n_g}{n_w} \right) p + t \right] = -\left[ \left( \frac{n_a}{n_w} \right) p + \left( \frac{n_a}{n_g} \right) t \right]
\]

(a) If the jelly fish is located 1.00 m (or 100 cm) in front of a 6.00 cm thick pane of glass, then \(p = +100\) cm and \(t = 6.00\) cm and the position of the final image relative to the glass-air interface is

\[
q = -\left[ \left( \frac{1.00}{1.333} \right) 100 \text{ cm} + \left( \frac{1.00}{1.50} \right) 6.00 \text{ cm} \right] = -79.0 \text{ cm} = -0.790 \text{ m}
\]

(b) If the thickness of the glass is negligible \((t \to 0)\), the distance of the final image from the glass-air interface is

\[
q = -\frac{n_a}{n_g} \left[ \left( \frac{n_g}{n_w} \right) p + 0 \right] = -\left( \frac{n_a}{n_w} \right) p = -\left( \frac{1.00}{1.333} \right) 100 \text{ cm} = -75.0 \text{ cm} = -0.750 \text{ m}
\]

so we see that the 6.00 cm thickness of the glass in part (a) made a 4.00 cm difference in the apparent position of the jelly fish.

(c) The thicker the glass, the greater the distance between the final image and the outer surface of the glass.

23.28 The wall of the aquarium (assumed to be of negligible thickness) is a plane \((R \to \infty)\) refracting surface separating water \((n_1 = 1.333)\) and air \((n_2 = 1.00)\). Thus, \(\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}\) gives the image position as \(q = -\frac{n_2}{n_1} p = -\frac{p}{1.333}\). When the object position changes by \(\Delta p\), the change in the image position is \(\Delta q = -\frac{\Delta p}{1.333}\). The apparent speed of the fish is then given by

\[
v_{\text{image}} = \frac{\Delta q}{\Delta t} = \frac{\Delta p/\Delta t}{1.333} = \frac{2.00 \text{ cm/s}}{1.333} = 1.50 \text{ cm/s}
\]
23.29 With \( R_1 = +2.00 \text{ cm} \) and \( R_2 = +2.50 \text{ cm} \), the lens maker’s equation gives the focal length as

\[
\frac{1}{f} = n - 1 \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = 1.50 - 1 \left( \frac{1}{2.00 \text{ cm}} - \frac{1}{2.50 \text{ cm}} \right) = 0.050 \text{ cm}^{-1}
\]

or

\[
f = \frac{1}{0.050 \text{ cm}^{-1}} = 20.0 \text{ cm}
\]

23.30 The lens maker’s equation is used to compute the focal length in each case.

(a) \[
\frac{1}{f} = n - 1 \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]
\]

\[
\frac{1}{f} = 1.44 - 1 \left[ \frac{1}{12.0 \text{ cm}} - \frac{1}{-18.0 \text{ cm}} \right] \quad f = 16.4 \text{ cm}
\]

(b) \[
\frac{1}{f} = 1.44 - 1 \left[ \frac{1}{18.0 \text{ cm}} - \frac{1}{-12.0 \text{ cm}} \right] \quad f = 16.4 \text{ cm}
\]

23.31 The focal length of a converging lens is positive, so \( f = +10.0 \text{ cm} \). The thin lens equation then yields a focal length of

\[
q = \frac{pf}{p - f} = \frac{p \cdot 10.0 \text{ cm}}{p - 10.0 \text{ cm}}
\]

(a) When \( p = +20.0 \text{ cm} \),

\[
q = \frac{20.0 \text{ cm} \cdot 10.0 \text{ cm}}{20.0 \text{ cm} - 10.0 \text{ cm}} = +20.0 \text{ cm} \quad \text{and} \quad M = -\frac{q}{p} = -\frac{20.0 \text{ cm}}{20.0 \text{ cm}} = -1.00
\]

so the image is located \( 20.0 \text{ cm} \) beyond the lens, is real \((q > 0)\), is inverted \((M < 0)\), and is the same size as the object \(|M| = 1.00\).

(b) When \( p = f = +10.0 \text{ cm} \), the object is at the focal point and no image is formed. Instead, parallel rays emerge from the lens.
(c) When \( p = 5.00 \text{ cm} \),

\[
q = \frac{5.00 \text{ cm} \cdot 10.0 \text{ cm}}{5.00 \text{ cm} - 10.0 \text{ cm}} = -10.0 \text{ cm}
\]
and

\[
M = \frac{-q}{p} = \frac{-10.0 \text{ cm}}{5.00 \text{ cm}} = +2.00
\]

so the image is located 10.0 cm in front of the lens, is virtual \( (q < 0) \), is upright \( (M > 0) \), and is the twice the size of the object \( |M| = 2.00 \).

23.32 (a) and (b) Your scale drawings should look similar to those given below:

![Figure (a)](image-a)

![Figure (b)](image-b)

A carefully drawn-to-scale version of Figure (a) should yield a real, inverted image that is located 20 cm in back of the lens and the same size as the object. Similarly, a carefully drawn-to-scale version of Figure (b) should yield an upright, virtual image located 10 cm in front of the lens and twice the size of the object.

(c) The accuracy of the graph depends on how accurately the ray diagrams are drawn. Sources of uncertainty: a parallel line from the tip of the object may not be exactly parallel; the focal points may not be exactly located; lines through the focal points may not be exactly the correct slope; the location of the intersection of two lines cannot be determined with complete accuracy.

23.33 From the thin lens equation, \( \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \), the image distance is found to be

\[
q = \frac{fp}{p-f} = \frac{-20.0 \text{ cm} \cdot p}{p - (-20.0 \text{ cm})} = \frac{-20.0 \text{ cm} \cdot p}{p + 20.0 \text{ cm}}
\]

(a) If \( p = 40.0 \text{ cm} \), then \( q = -13.3 \text{ cm} \) and \( M = \frac{-q}{p} = \frac{-13.3 \text{ cm}}{40.0 \text{ cm}} = +1/3 \)

The image is virtual, upright, and 13.3 cm in front of the lens.
(b) If \( p = 20.0 \text{ cm} \), then \( q = -10.0 \text{ cm} \) and

\[
M = -\frac{q}{p} = -\frac{-10.0 \text{ cm}}{20.0 \text{ cm}} = +1/2
\]

The image is virtual, upright, and 10.0 cm in front of the lens.

(c) When \( p = 10.0 \text{ cm} \), \( q = -6.67 \text{ cm} \) and \( M = -\frac{q}{p} = -\frac{-6.67 \text{ cm}}{10.0 \text{ cm}} = +2/3 \)

The image is virtual, upright, and 6.67 cm in front of the lens.

23.34 (a) and (b) Your scale drawings should look similar to those given below:

![Figure (a)]

![Figure (b)]

A carefully drawn-to-scale version of Figure (a) should yield an upright, virtual image located 13.3 cm in front of the lens and one-third the size of the object. Similarly, a carefully drawn-to-scale version of Figure (b) should yield an upright, virtual image located 6.7 cm in front of the lens and two-thirds the size of the object.

(c) The results of the graphical solution are consistent with the algebraic answers found in problem 23.33, allowing for small deviations due to uncertainties in measurement. Graphical answers may vary, depending on the size of the graph paper and accuracy of the drawing.
23.35 (a) The real image case is shown in the ray diagram. Notice that \( p + q = 12.9 \text{ cm} \), or \( q = 12.9 \text{ cm} - p \). The thin lens equation, with \( f = 2.44 \text{ cm} \), then gives

\[
\frac{1}{p} + \frac{1}{12.9 \text{ cm} - p} = \frac{1}{2.44 \text{ cm}}
\]

or

\[
p^2 - 12.9 \text{ cm} \; p + 31.5 \text{ cm}^2 = 0
\]

Using the quadratic formula to solve gives

\[
p = 9.63 \text{ cm} \; \text{or} \; p = 3.27 \text{ cm}
\]

Both are valid solutions for the real image case.

(b) The virtual image case is shown in the second diagram. Note that in this case, \( q = -12.9 \text{ cm} + p \), so the thin lens equation gives

\[
\frac{1}{p} - \frac{1}{12.9 \text{ cm} + p} = \frac{1}{2.44 \text{ cm}}
\]

or

\[
p^2 + 12.9 \text{ cm} \; p - 31.5 \text{ cm}^2 = 0
\]

The quadratic formula then gives \( p = 2.10 \text{ cm} \; \text{or} \; p = -15.0 \text{ cm} \).

Since the object is real, the negative solution must be rejected leaving \( p = 2.10 \text{ cm} \).

23.36 We must first realize that we are looking at an upright, magnified, virtual image. Thus, we have a real object located between a converging lens and its front-side focal point, so \( q < 0, p > 0, \) and \( f > 0 \).

The magnification is \( M = -\frac{q}{p} = +2 \), giving \( q = -2p \). Then, from the thin lens equation,

\[
\frac{1}{p} - \frac{1}{2p} = +\frac{1}{2p} = \frac{1}{f} \; \text{or} \; f = 2p = 2 \; 2.84 \text{ cm} = 5.68 \text{ cm}
\]
23.37 It is desired to form a magnified, real image on the screen using a single thin lens. To do this, a converging lens must be used and the image will be inverted. The magnification then gives

\[ M = \frac{h'}{h} = -\frac{1.80 \text{ m}}{24.0 \times 10^{-3} \text{ m}} = -\frac{q}{p}, \text{ or } q = 75.0p \]

Also, we know that \( p + q = 3.00 \text{ m} \). Therefore, \( p + 75.0p = 3.00 \text{ m} \) giving

(b) \( p = \frac{3.00 \text{ m}}{76.0} = 3.95 \times 10^{-2} \text{ m} = 39.5 \text{ mm} \)

(a) The thin lens equation then gives

\[ \frac{1}{p} + \frac{1}{75.0p} = \frac{76.0}{75.0p} = \frac{1}{f} \]

or \( f = \left( \frac{75.0}{76.0} \right)p = \left( \frac{75.0}{76.0} \right)39.5 \text{ mm} = 39.0 \text{ mm} \)

23.38 To have a magnification of \( M = -q/p = +3.00 \), it is necessary that \( q = -3.00p \). The thin lens equation, with \( f = +18.0 \text{ cm} \) for the convergent convex lens, gives the required object distance as

\[ \frac{1}{p} - \frac{1}{3.00p} = \frac{2}{3.00p} = \frac{1}{18.0 \text{ cm}} \text{ or } p = \frac{2 \times 18.0 \text{ cm}}{3.00} = 12.0 \text{ cm} \]

23.39 Since the light rays incident to the first lens are parallel, \( p_1 = \infty \) and the thin lens equation gives \( q_1 = f_1 = -10.0 \text{ cm} \).

The virtual image formed by the first lens serves as the object for the second lens, so \( p_2 = 30.0 \text{ cm} + |q_1| = 40.0 \text{ cm} \). If the light rays leaving the second lens are parallel, then \( q_2 = \infty \) and the thin lens equation gives \( f_2 = p_2 = 40.0 \text{ cm} \).

23.40 (a) Solving the thin lens equation for the image distance \( q \) gives

\[ \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{p - f}{pf} \text{ or } q = \frac{pf}{p - f} \]
(b) For a real object, \( p > 0 \) and \( p = |p| \). Also, for a diverging lens, \( f < 0 \) and \( f = -|f| \).

The result of part (a) then becomes

\[
q = \frac{|p| - |f|}{|p| - |f|} = \frac{|p| |f|}{|p| + |f|}
\]

Thus, we see that \( q < 0 \) for all numeric values of \( |p| \) and \( |f| \). Since negative image distances mean virtual images, we conclude that a diverging lens will always form virtual images of real objects.

(c) For a real object, \( p > 0 \) and \( p = |p| \). Also, for a converging lens, \( f > 0 \) and \( f = |f| \).

The result of part (a) then becomes

\[
q = \frac{|p| |f|}{|p| - |f|} > 0 \quad \text{if} \quad |p| - |f| > 0
\]

Since \( q \) must be positive for a real image, we see that a converging lens will form real images of real objects only when \( |p| > |f| \) (or \( |p| > |f| \) since both \( p \) and \( f \) are positive in this situation).

23.41 The thin lens equation gives the image position for the first lens as

\[
q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{30.0 \text{ cm} \cdot 15.0 \text{ cm}}{30.0 \text{ cm} - 15.0 \text{ cm}} = +30.0 \text{ cm}
\]

and the magnification by this lens is \( M_1 = -\frac{q_1}{p_1} = -\frac{30.0 \text{ cm}}{30.0 \text{ cm}} = -1.00 \).

The real image formed by the first lens serves as the object for the second lens, so \( p_2 = 40.0 \text{ cm} - q_1 = +10.0 \text{ cm} \). Then, the thin lens equation gives

\[
q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{10.0 \text{ cm} \cdot 15.0 \text{ cm}}{10.0 \text{ cm} - 15.0 \text{ cm}} = -30.0 \text{ cm}
\]

and the magnification by the second lens is

\[
M_2 = -\frac{q_2}{p_2} = -\frac{-30.0 \text{ cm}}{10.0 \text{ cm}} = +3.00
\]

Thus, the final, virtual image is located \( 30.0 \text{ cm} \) in front of the second lens.
and the overall magnification is \( M = M_1 M_2 = -1.00 \times 3.00 = -3.00 \)

23.42 (a) With \( p_1 = +15.0 \text{ cm} \), the thin lens equation gives the position of the image formed by the first lens as

\[
q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{15.0 \text{ cm} \times 10.0 \text{ cm}}{15.0 \text{ cm} - 10.0 \text{ cm}} = +30.0 \text{ cm}
\]

This image serves as the object for the second lens, with an object distance of \( p_2 = 10.0 \text{ cm} - q_1 = 10.0 \text{ cm} - 30.0 \text{ cm} = -20.0 \text{ cm} \) (a virtual object). If the image formed by this lens is at the position of \( O \), the image distance is

\[
q_2 = -10.0 \text{ cm} + p_1 = -10.0 \text{ cm} + 15.0 \text{ cm} = -25.0 \text{ cm}
\]

The thin lens equation then gives the focal length of the second lens as

\[
f_2 = \frac{p_2 q_2}{p_2 + q_2} = \frac{-20.0 \text{ cm} \times -25.0 \text{ cm}}{-20.0 \text{ cm} - 25.0 \text{ cm}} = -11.1 \text{ cm}
\]

(b) The overall magnification is

\[
M = M_1 M_2 = \left( \frac{-q_1}{p_1} \right) \left( \frac{-q_2}{p_2} \right) = \left( \frac{-30.0 \text{ cm}}{15.0 \text{ cm}} \right) \left( \frac{-25.0 \text{ cm}}{-20.0 \text{ cm}} \right) = +2.50
\]

(c) Since \( q_2 < 0 \), the final image is \( \text{virtual} \); and since \( M > 0 \), it is \( \text{upright} \).
23.43 From the thin lens equation, \( q_i = \frac{p_i f_i}{p_i - f_i} = \frac{4.00 \text{ cm} \times 8.00 \text{ cm}}{4.00 \text{ cm} - 8.00 \text{ cm}} = -8.00 \text{ cm} \)

The magnification by the first lens is \( M_1 = -\frac{q_i}{p_i} = -\frac{-8.00 \text{ cm}}{4.00 \text{ cm}} = +2.00 \)

The virtual image formed by the first lens is the object for the second lens, so \( p_2 = 6.00 \text{ cm} + |q_i| = +14.0 \text{ cm} \) and the thin lens equation gives

\[
q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{14.0 \text{ cm} \times -16.0 \text{ cm}}{14.0 \text{ cm} - -16.0 \text{ cm}} = -7.47 \text{ cm}
\]

The magnification by the second lens is \( M_2 = -\frac{q_2}{p_2} = -\frac{-7.47 \text{ cm}}{14.0 \text{ cm}} = +0.533 \), so the overall magnification is \( M = M_1 M_2 = +2.00 \times +0.533 = +1.07 \)

The position of the final image is [7.47 cm in front of the second lens], and its height is

\[
h' = M h = +1.07 \times 1.00 \text{ cm} = 1.07 \text{ cm}
\]

Since \( M > 0 \), the final image is [upright]; and since \( q_2 < 0 \), this image is [virtual]

23.44 (a) We start with the final image and work backward. From Figure P23.44, observe that \( q_2 = -50.0 \text{ cm} - 31.0 \text{ cm} = -19.0 \text{ cm} \). The thin lens equation then gives

\[
p_2 = \frac{q_2 f_2}{q_2 - f_2} = \frac{-19.0 \text{ cm} \times 20.0 \text{ cm}}{-19.0 \text{ cm} - 20.0 \text{ cm}} = +9.74 \text{ cm}
\]

The image formed by the first lens serves as the object for the second lens and is located 9.74 cm in front of the second lens.

Thus, \( q_i = 50.0 \text{ cm} - 9.74 \text{ cm} = 40.3 \text{ cm} \) and the thin lens equation gives

\[
p_i = \frac{q_i f_i}{q_i - f_i} = \frac{40.3 \text{ cm} \times 10.0 \text{ cm}}{40.3 \text{ cm} - 10.0 \text{ cm}} = +13.3 \text{ cm}
\]

The original object should be located [13.3 cm] in front of the first lens.
(b) The overall magnification is

\[ M = M_1 M_2 = \left( \frac{q_1}{p_1} \right) \left( \frac{q_2}{p_2} \right) = \left( \frac{40.3 \text{ cm}}{13.3 \text{ cm}} \right) \left( \frac{-19.0 \text{ cm}}{9.74 \text{ cm}} \right) = -5.90 \]

(c) Since \( M < 0 \), the final image is inverted; and since \( q_2 < 0 \), it is virtual.

23.45 **Note:** Final answers to this problem are highly sensitive to round-off error. To avoid this, we retain extra digits in intermediate answers and round only the final answers to the correct number of significant figures.

Since the final image is to be real and in the film plane, \( q_2 = +d \)

Then, the thin lens equation gives \( p_2 = \frac{q_2 f_2}{q_2 - f_2} = \frac{d}{d - 13.0 \text{ cm}} \)

From Figure P23.45, observe that \( d < 12.0 \text{ cm} \). The above result then shows that \( p_2 < 0 \), so the object for the second lens will be a virtual object.

The object of the second lens \( L_2 \) is the image formed by the first lens \( L_1 \), so

\[ q_i = 12.0 \text{ cm} - d - p_2 = 12.0 \text{ cm} - d \left( 1 + \frac{13.0 \text{ cm}}{d - 13.0 \text{ cm}} \right) = 12.0 \text{ cm} - \frac{d^2}{d - 13.0 \text{ cm}} \]

If \( d = 5.00 \text{ cm} \), then \( q_i = +15.125 \text{ cm} \); and when \( d = 10.0 \text{ cm} \), \( q_i = +45.333 \text{ cm} \)

From the thin lens equation, \( p_1 = \frac{-q_i f_1}{q_i - f_1} = \frac{q_i}{q_i - 15.0 \text{ cm}} \)

When \( q_i = +15.125 \text{ cm} \) \( d = 5.00 \text{ cm} \), then \( p_1 = 1.82 \times 10^3 \text{ cm} = 18.2 \text{ m} \)

When \( q_i = +45.333 \text{ cm} \) \( d = 10.0 \text{ cm} \), then \( p_1 = 22.4 \text{ cm} = 0.224 \text{ m} \)

Thus, the range of focal distances for this camera is \( 0.224 \text{ m} \) to \( 18.2 \text{ m} \).
23.46  (a) From the thin lens equation, the image distance for the first lens is

\[ q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{15.0 \text{ cm} \quad 10.0 \text{ cm}}{15.0 \text{ cm} - 10.0 \text{ cm}} = +30.0 \text{ cm} \]

(b) With \( q_1 = +30.0 \text{ cm} \), the image of the first lens is located 30.0 cm in back of that lens. Since the second lens is only 10.0 cm beyond the first lens, this means that the first lens is trying to form its image at a location 20.0 cm beyond the second lens.

(c) The image the first lens forms (or would form if allowed to do so) serves as the object for the second lens. Considering the answer to part (b) above, we see that this will be a virtual object, with object distance \( p_2 = -20.0 \text{ cm} \).

(d) From the thin lens equation, the image distance for the second lens is

\[ q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{-20.0 \text{ cm} \quad 5.00 \text{ cm}}{-20.0 \text{ cm} - 5.00 \text{ cm}} = +4.00 \text{ cm} \]

(e) \( M_1 = -\frac{q_1}{p_1} = -\frac{30.0 \text{ cm}}{15.0 \text{ cm}} = -2.00 \)

(f) \( M_2 = -\frac{q_2}{p_2} = -\frac{4.00 \text{ cm}}{-20.0 \text{ cm}} = +0.200 \)

(g) \( M = M_1 M_2 = -2.00 \times +0.200 = -0.400 \)

(h) Since \( q_2 > 0 \), the final image is \( \text{real} \), and since \( M < 0 \), that image is \( \text{inverted} \).

23.47 Since \( q = +8.00 \text{ cm when } p = +10.0 \text{ cm} \), we find that

\[ \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}} + \frac{1}{8.00 \text{ cm}} = \frac{18.0}{80.0 \text{ cm}} \]

Then, when \( p = 20.0 \text{ cm} \),

\[ \frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{18.0}{80.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = \frac{18.0 - 4.00}{80.0 \text{ cm}} = \frac{14.0}{80.0 \text{ cm}} \]

or \( q = \frac{80.0 \text{ cm}}{14.0} = +5.71 \text{ cm} \)

Thus, a \( \text{real} \) image is formed 5.71 cm in front of the mirror.
23.48 (a) We are given that \( p=5f \), with both \( p \) and \( f \) being positive. The thin lens equation then gives

\[
q = \frac{pf}{p-f} = \frac{5f}{5f-f} = \frac{5f}{4f} = \frac{5}{4}
\]

(b) \( M = -\frac{q}{p} = -\frac{5f/4}{5f} = -\frac{1}{4} \)

(c) Since \( q>0 \), the image is real. Because \( M<0 \), the image is inverted. Since the object is real, it is located in front of the lens, and with \( q>0 \), the image is located in back of the lens. Thus, the image is on the opposite side of the lens from the object.

23.49 Since the object is very distant \( p \to \infty \), the image distance equals the focal length, or \( q=+50.0 \text{ mm} \). Now consider two rays that pass undeviated through the center of the thin lens to opposite sides of the image as shown in the sketch below.

From the sketch, observe that

\[
\tan \left( \frac{\alpha}{2} \right) = \frac{1}{2} \frac{35.0 \text{ mm}}{50.0 \text{ mm}} = 0.350
\]

Thus, the angular width of the image is

\[
\alpha = 2\tan^{-1} 0.350 = 38.6^\circ
\]

23.50 (a) Using the sign convention from Table 23.2, the radii of curvature of the surfaces are \( R_1 = -15.0 \text{ cm} \) and \( R_2 = +10.0 \text{ cm} \). The lens maker’s equation then gives

\[
\frac{1}{f} = n-1 \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = 1.50 - 1 \left( \frac{1}{-15.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} \right) \text{ or } f = -12.0 \text{ cm}
\]

(b) If \( p \to \infty \), then \( q = f = -12.0 \text{ cm} \)

The thin lens equation gives, \( q = \frac{pf}{p-f} = \frac{p}{p+12.0 \text{ cm}} \) and the following results:
(c) If \( p = 3|f| = +36.0 \text{ cm} \), \( q = -9.00 \text{ cm} \)

(d) If \( p = |f| = +12.0 \text{ cm} \), \( q = -6.00 \text{ cm} \)

(e) If \( p = |f|/2 = +6.00 \text{ cm} \), \( q = -4.00 \text{ cm} \)

23.51 As light passes left-to-right through the lens, the image position is given by

\[
q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{100 \text{ cm} \cdot 80.0 \text{ cm}}{100 \text{ cm} - 80.0 \text{ cm}} = +400 \text{ cm}
\]

This image serves as an object for the mirror with an object distance of \( p_2 = 100 \text{ cm} - q_1 = -300 \text{ cm} \) (virtual object). From the mirror equation, the position of the image formed by the mirror is

\[
q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{-300 \text{ cm} \cdot -50.0 \text{ cm}}{-300 \text{ cm} - -50.0 \text{ cm}} = -60.0 \text{ cm}
\]

This image is the object for the lens as light now passes through it going right-to-left. The object distance for the lens is \( p_3 = 100 \text{ cm} - q_2 = 100 \text{ cm} - -60.0 \text{ cm} \), or \( p_3 = 160 \text{ cm} \). From the thin lens equation,

\[
q_3 = \frac{p_3 f_3}{p_3 - f_3} = \frac{160 \text{ cm} \cdot 80.0 \text{ cm}}{160 \text{ cm} - 80.0 \text{ cm}} = +160 \text{ cm}
\]

Thus, the final image is located \( 160 \text{ cm to the left of the lens} \)

The overall magnification is \( M = M_1 M_2 M_3 = \left( -\frac{q_1}{p_1} \right) \left( -\frac{q_2}{p_2} \right) \left( -\frac{q_3}{p_3} \right) \), or

\[
M = \begin{pmatrix} -400 \text{ cm} \\ 100 \text{ cm} \end{pmatrix} \begin{pmatrix} -60.0 \text{ cm} \\ -300 \text{ cm} \end{pmatrix} \begin{pmatrix} -160 \text{ cm} \\ -160 \text{ cm} \end{pmatrix} = -0.800
\]

Since \( M < 0 \), the final image is \( \text{inverted} \)
23.52 Since the object is midway between the lens and mirror, the object distance for the mirror is $p_1 = +12.5$ cm. The mirror equation gives the image position as

$$\frac{1}{q_1} = \frac{2}{R} - \frac{1}{p_1} = \frac{2}{20.0\text{ cm}} - \frac{1}{12.5\text{ cm}} = \frac{5 - 4}{50.0\text{ cm}} = \frac{1}{50.0\text{ cm}}, \text{ or } q_1 = +50.0\text{ cm}$$

This image serves as the object for the lens, so $p_2 = 25.0\text{ cm} - q_1 = -25.0\text{ cm}$. Note that since $p_2 < 0$, this is a virtual object. The thin lens equation gives the image position for the lens as

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{-25.0\text{ cm} - -16.7\text{ cm}}{-25.0\text{ cm} - -16.7\text{ cm}} = -50.3\text{ cm}$$

Since $q_2 < 0$, this is a virtual image that is located 50.3 cm in front of the lens or 25.3 cm behind the mirror. The overall magnification is

$$M = M_1 M_2 = \left( -\frac{q_1}{p_1} \right) \left( -\frac{q_2}{p_2} \right) = \left( -\frac{50.0\text{ cm}}{12.5\text{ cm}} \right) \left[ -\frac{-50.3\text{ cm}}{-25.0\text{ cm}} \right] = +8.05$$

Since $M > 0$, the final image is upright.

23.53 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light’s exit from the curved surface, for which $R = -6.00$ cm.

The incident rays are parallel, so $p = \infty$.

Then, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $0 + \frac{1.00}{q} = \frac{1.00 - 1.56}{-6.00\text{ cm}}$

from which $q = 10.7$ cm
23.54  (a) The thin lens equation gives the image distance for the first lens as

\[ q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{40.0 \text{ cm} - 20.0 \text{ cm}}{40.0 \text{ cm} - 20.0 \text{ cm}} = 40.0 \text{ cm} \]

The magnification by this lens is then \( M_1 = -\frac{q_1}{p_1} = -\frac{40.0 \text{ cm}}{40.0 \text{ cm}} = -1.00 \)

The real image formed by the first lens is the object for the second lens. Thus, \( p_2 = 50.0 \text{ cm} - q_1 = +10.0 \text{ cm} \) and the thin lens equation gives

\[ q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{10.0 \text{ cm} - 5.00 \text{ cm}}{10.0 \text{ cm} - 5.00 \text{ cm}} = 10.0 \text{ cm} \]

The final image is \( 10.0 \text{ cm in back of the second lens} \)

(b) The magnification by the second lens is \( M_2 = -\frac{q_2}{p_2} = -\frac{10.0 \text{ cm}}{10.0 \text{ cm}} = -1.00 \), so the overall magnification is \( M = M_1 M_2 = -1.00 \times -1.00 = +1.00 \). Since this magnification has a value of unity, the final image is the same size as the original object, or \( h' = M h_1 = +1.00 \times 2.00 \text{ cm} = 2.00 \text{ cm} \)

The image distance for the second lens is positive, so the final image is \( \text{real} \).

(c) When the two lenses are in contact, the focal length of the combination is

\[ \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{20.0 \text{ cm}} + \frac{1}{5.00 \text{ cm}} \], or \( f = 4.00 \text{ cm} \)

The image position is then

\[ q = \frac{p f}{p - f} = \frac{5.00 \text{ cm} - 4.00 \text{ cm}}{5.00 \text{ cm} - 4.00 \text{ cm}} = +20.0 \text{ cm} \]
23.55 With light going through the piece of glass from left to right, the radius of the first surface is positive and that of the second surface is negative according to the sign convention of Table 23.2. Thus, \( R_1 = 2.00 \text{ cm} \) and \( R_2 = -4.00 \text{ cm} \).

Applying \( \frac{n_1 + n_2}{p} = \frac{n_2 - n_1}{q} \frac{1}{R} \) to the first surface gives

\[
\frac{1.00}{1.00 \text{ cm}} + \frac{1.50}{q_1} = \frac{1.50 - 1.00}{+2.00 \text{ cm}}
\]

which yields \( q_1 = -2.00 \text{ cm} \). The first surface forms a virtual image 2.00 cm to the left of that surface and 16.0 cm to the left of the second surface.

The image formed by the first surface is the object for the second surface, so \( p_2 = +16.0 \text{ cm} \) and \( \frac{n_1 + n_2}{p} = \frac{n_2 - n_1}{q} \frac{1}{R} \) gives

\[
\frac{1.50}{16.0 \text{ cm}} + \frac{1.00}{q_2} = \frac{1.00 - 1.50}{-4.00 \text{ cm}} \text{ or } q_2 = +32.0 \text{ cm}
\]

The final image formed by the piece of glass is a real image located 32.0 cm to the right of the second surface.

23.56 Consider an object \( O_1 \) at distance \( p_1 \) in front of the first lens. The thin lens equation gives the image position for this lens as

\[
\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1}.
\]

The image, \( I_1 \), formed by the first lens serves as the object, \( O_2 \), for the second lens. With the lenses in contact, this will be a virtual object if \( I_1 \) is real and will be a real object if \( I_1 \) is virtual. In either case, if the thicknesses of the lenses may be ignored,

\[
p_2 = -q_1 \text{ and } \frac{1}{p_2} = -\frac{1}{q_1} = -\frac{1}{f_1} + \frac{1}{p_1}
\]

Applying the thin lens equation to the second lens, \( \frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \) becomes
\[-\frac{1}{f_1} + \frac{1}{p_1} + \frac{1}{q_2} = \frac{1}{f_2} \text{ or } \frac{1}{p_1} + \frac{1}{q_2} = \frac{1}{f_1} + \frac{1}{f_2}\]

Observe that this result is a thin lens type equation relating the position of the original object \(O_1\) and the position of the final image \(I_2\) formed by this two lens combination. Thus, we see that we may treat two thin lenses in contact as a single lens having a focal length, \(f\), given by

\[\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}\]

23.57 From the thin lens equation, the image distance for the first lens is

\[q_i = \frac{p_1 f_1}{p_1 - f_1} = \frac{40.0 \text{ cm}}{40.0 \text{ cm} - 30.0 \text{ cm}} = +120 \text{ cm}\]

and the magnification by this lens is \(M_1 = -\frac{q_i}{p_1} = -\frac{120 \text{ cm}}{40.0 \text{ cm}} = -3.00\)

The real image formed by the first lens serves as the object for the second lens, with object distance of \(p_2 = 110 \text{ cm} \) \(-q_i = -10.0 \text{ cm}\) (a virtual object). The thin lens equation gives the image distance for the second lens as

\[q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{-10.0 \text{ cm}}{-10.0 \text{ cm} - f_2}\]

(a) If \(f_2 = -20.0 \text{ cm}\), then \(q_2 = +20.0 \text{ cm}\) and the magnification by the second lens is \(M_2 = -\frac{q_2}{p_2} = -\frac{20.0 \text{ cm}}{-10.0 \text{ cm}} = +2.00\)

The final image is located [20.0 cm to the right of the second lens] and the overall magnification is \(M = M_1 M_2 = -3.00 \times +2.00 = -6.00\)

(b) Since \(M < 0\), the final image is [inverted]
(c) If \( f_2 = +20.0 \text{ cm} \), then \( q_2 = +6.67 \text{ cm} \)

and \( M_2 = \frac{-q_2}{p_2} = -\frac{6.67 \text{ cm}}{-10.0 \text{ cm}} = +0.667 \)

The final image is \( 6.67 \text{ cm} \) to the right of the second lens

and the overall magnification is \( M = M_1M_2 = -3.00 \cdot +0.667 = \boxed{-2.00} \)

Since \( M < 0 \), the final image is inverted.

23.58 The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (that is, parallel rays leave this mirror). The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror. Thus, the image is real, inverted, and actual size.

For the upper mirror:

\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \Rightarrow \quad \frac{1}{7.50 \text{ cm}} + \frac{1}{q_1} = \frac{1}{7.50 \text{ cm}} ; \quad q_1 = \infty
\]

For the lower mirror:

\[
\frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50 \text{ cm}} ; \quad q_2 = 7.50 \text{ cm}
\]

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

23.59 (a) The lens maker’s equation, \( \frac{1}{f} = n - 1 \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \), gives

\[
\frac{1}{5.00 \text{ cm}} = n - 1 \left( \frac{1}{9.00 \text{ cm}} - \frac{1}{-11.0 \text{ cm}} \right)
\]

which simplifies to \( n = 1 + \frac{1}{5.00} \left( \frac{99.0}{11.0 + 9.00} \right) = 1.99 \)
(b) As light passes from left to right through the lens, the thin lens equation gives the image distance as

\[ q_1 = \frac{p_1 f}{p_1 - f} = \frac{8.00 \text{ cm} \times 5.00 \text{ cm}}{8.00 \text{ cm} - 5.00 \text{ cm}} = +13.3 \text{ cm} \]

This image formed by the lens serves as an object for the mirror with object distance \( p_2 = 20.0 \text{ cm} - q_1 = +6.67 \text{ cm} \). The mirror equation then gives

\[ q_2 = \frac{p_2 R}{2 p_2 - R} = \frac{6.67 \text{ cm} \times 8.00 \text{ cm}}{2 \times 6.67 \text{ cm} - 8.00 \text{ cm}} = +10.0 \text{ cm} \]

This real image, formed 10.0 cm to the left of the mirror, serves as an object for the lens as light passes through it from right to left. The object distance is \( p_3 = 20.0 \text{ cm} - q_2 = +10.0 \text{ cm} \), and the thin lens equation gives

\[ q_3 = \frac{p_3 f}{p_3 - f} = \frac{10.0 \text{ cm} \times 5.00 \text{ cm}}{10.0 \text{ cm} - 5.00 \text{ cm}} = +10.0 \text{ cm} \]

The final image is located 10.0 cm to the left of the lens and its overall magnification is

\[ M = M_1 M_2 M_3 = \left( -\frac{q_1}{p_1} \right) \left( -\frac{q_2}{p_2} \right) \left( -\frac{q_3}{p_3} \right) = \left( -\frac{13.3}{8.00} \right) \left( -\frac{6.67}{10.0} \right) \left( -\frac{10.0}{10.0} \right) = -2.50 \]

(c) Since \( M < 0 \), the final image is inverted.

23.60 From the thin lens equation, the object distance is \( p = \frac{-q f}{q - f} \)

(a) If \( q = +4f \), then \( p = \frac{4f f}{4f - f} = \frac{4f}{3} \)

(b) When \( q = -3f \), we find \( p = \frac{-3f f}{-3f - f} = \frac{3f}{4} \)

(c) In case (a), \( M = -\frac{q}{p} = -\frac{4f}{4f/3} = -3 \)

and in case (b), \( M = -\frac{q}{p} = -\frac{3f}{3f/4} = 4 \)
23.61 If \( R_1 = -3.00 \text{ m} \) and \( R_2 = -6.00 \text{ m} \), the focal length is given by

\[
\frac{1}{f} = \left( \frac{n_1}{n_2} - 1 \right) \left( \frac{1}{-3.00 \text{ m}} + \frac{1}{6.00 \text{ m}} \right) = \left( \frac{n_1 - n_2}{n_2} \right) \left( \frac{-1}{6.00 \text{ m}} \right)
\]

or

\[
f = \frac{6.00 \text{ m} \ n_2}{n_2 - n_1}
\]

(a) If \( n_1 = 1.50 \) and \( n_2 = 1.00 \), then

\[
f = \frac{6.00 \text{ m} \ 1.00}{1.00 - 1.50} = -12.0 \text{ m}
\]

The thin lens equation gives

\[
q = \frac{pf}{p-f} = \frac{10.0 \text{ m} \ -12.0 \text{ m}}{10.0 \text{ m} + 12.0 \text{ m}} = -5.45 \text{ m}
\]

A virtual image is formed 5.45 m to the left of the lens.

(b) If \( n_1 = 1.50 \) and \( n_2 = 1.33 \), the focal length is

\[
f = \frac{6.00 \text{ m} \ 1.33}{1.33 - 1.50} = -46.9 \text{ m}
\]

and

\[
q = \frac{pf}{p-f} = \frac{10.0 \text{ m} \ -46.9 \text{ m}}{10.0 \text{ m} + 46.9 \text{ m}} = -8.24 \text{ m}
\]

The image is located 8.24 m to the left of the lens.

(c) When \( n_1 = 1.50 \) and \( n_2 = 2.00 \), 

\[
f = \frac{6.00 \text{ m} \ 2.00}{2.00 - 1.50} = +24.0 \text{ m}
\]

and

\[
q = \frac{pf}{p-f} = \frac{10.0 \text{ m} \ 24.0 \text{ m}}{10.0 \text{ m} - 24.0 \text{ m}} = -17.1 \text{ m}
\]

The image is 17.1 m to the left of the lens.

(d) Observe from Equation [1] that \( f < 0 \) if \( n_1 > n_2 \) and \( f > 0 \) when \( n_1 < n_2 \). Thus, a diverging lens can be changed to converging by surrounding it with a medium whose index of refraction exceeds that of the lens material.
23.62 The inverted image is formed by light that leaves the object and goes directly through the lens, never having reflected from the mirror. For the formation of this inverted image, we have

\[ M = -\frac{q_i}{p_i} = -1.50 \quad \text{giving} \quad q_i = +1.50 \, p_i \]

The thin lens equation then gives

\[ \frac{1}{p_i} + \frac{1}{1.50 \, p_i} = \frac{1}{10.0 \, \text{cm}} \quad \text{or} \quad p_i = 10.0 \, \text{cm} \left(1 + \frac{1}{1.50}\right) = 16.7 \, \text{cm} \]

The upright image is formed by light that passes through the lens after reflecting from the mirror. The object for the lens in this upright image formation is the image formed by the mirror. In order for the lens to form the upright image at the same location as the inverted image, the image formed by the mirror must be located at the position of the original object (so the object distances, and hence image distances, are the same for both the inverted and upright images formed by the lens). Therefore, the object distance and the image distance for the mirror are equal, and their common value is

\[ q_{\text{mirror}} = p_{\text{mirror}} = 40.0 \, \text{cm} - p_i = 40.0 \, \text{cm} - 16.7 \, \text{cm} = +23.3 \, \text{cm} \]

The mirror equation, \( \frac{1}{f_{\text{mirror}}} = \frac{1}{23.3 \, \text{cm}} + \frac{1}{23.3 \, \text{cm}} = \frac{-2}{23.3 \, \text{cm}} \) or \( f_{\text{mirror}} = \frac{23.3 \, \text{cm}}{2} = +11.7 \, \text{cm} \)

23.63. (a) The lens maker’s equation for a lens made of material with refractive index \( n_1 = 1.55 \) and immersed in a medium having refractive index \( n_2 \) is

\[ \frac{1}{f} = \left(\frac{n_1}{n_2} - 1\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1.55 - n_2}{n_2}\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \]

Thus, when the lens is in air, we have \( \frac{1}{f_{\text{air}}} = \left(\frac{1.55 - 1.00}{1.00}\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \) \[ \text{[1]} \]

and when it is immersed in water, \( \frac{1}{f_{\text{water}}} = \left(\frac{1.55 - 1.33}{1.33}\right) \left(\frac{1}{R_1} - \frac{1}{R_2}\right) \) \[ \text{[2]} \]

\[ \frac{f_{\text{water}}}{f_{\text{air}}} = \left( \frac{1.33}{1.00} \right) \left( \frac{1.55 - 1.00}{1.55 - 1.33} \right) = 1.33 \left( \frac{0.55}{0.22} \right) \]

If \( f_{\text{air}} = 79.0 \text{ cm} \), the focal length when immersed in water is

\[ f_{\text{water}} = 79.0 \text{ cm } 1.33 \left( \frac{0.55}{0.22} \right) = 263 \text{ cm} \]

(b) The focal length for a mirror is determined by the law of reflection, which is independent of the material of which the mirror is made and of the surrounding medium. Thus, the focal length depends only on the radius of curvature and not on the material making up the mirror or the surrounding medium. This means that, for the mirror,

\[ f_{\text{water}} = f_{\text{air}} = 79.0 \text{ cm} \]